Learning from Giants' Mistakes in Cataract Surgery Fedorov 67 vs Fyodorov 75

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Abstract

One of the most influential papers in the history of cataract surgery is Fedorov and Kolinko's 1967 paper. All so called third and fourth generation intraocular lens calculation formulas implemented to this day are based on it. We show that the actual expression given in the 1967 article is incorrect and was subsequently corrected by Fedorov and colleagues in 1975. We show the source of error and comment on implications for modern day intraocular power calculations for cataract surgery.

Introduction

Slava Fedorov's contributions to Ophthalmology are profound and lasting. In a paper with Albina Kolinko in 1967 they attempted a rational theoretical approach to the computation of intraocular lens (IOL) power based on biometry of the patient's eye[1]. The paper was written in Russian and was followed by a few similar attempts by mainly European ophthalmologists. In 1975, together with Galin and Linksz, Fedorov (Fyodorov) contributed a similar article and computation to the English literature[2]. This made the approach more accessible to a wider readership and was followed by the (re)introduction of regression methods exemplified by SRK[3,4] and somewhat later, by a realization that theoretical methods were foundational and need be adopted albeit with a grain of tweaking! This is represented by some of the major "third generation" or "vergence based" formulas such as Holladay1, SRKT and HofferQ[5,6,7], all of which cite at least one of the Fyodorov and colleague's papers and are in turn some of the most cited and implemented papers in the IOL power calculation literature.

To our knowledge, there is no systematic comparison that has been attempted between the 1967 and 1975 versions of Fedorov and colleagues' articles. In fact, a number of papers have cited both the 1967 and 1975 articles without mention of any discrepancy[4,6,8]. Others will cite only one[9] or the other[3,10,11]. Some will mention a date and cite the article from the other date[12]. Yet others, surprisingly, cite neither[13]. In the present work we start by a direct comparison of the key formulas and their differences including numeric estimates and clinical implications. We follow with an analysis of the original sources and the sources of errors and draw conclusions as to lessons to be learned.

The 1967 and 1975 formulas of Fedorov and colleagues are different. One is incorrect.

1967 version[1]

$$D_{IOL} = \frac{n - D_{\rho}\left(l - k + \frac{k}{n}\right)}{(l-k)\left(1 - \frac{kD_{\rho}}{n}\right)} \qquad (F67)$$

1975 version[2]

$$D_p = \frac{n - aD_c}{(a-k)\left(1 - \frac{kD_c}{n}\right)} \qquad (F75)$$

where the symbols used in the corresponding articles and in our presentation are given in tables 1 and 2 where ACD = Anterior Chamber Depth and ELP = Effective Lens Position.

Power	F67	F75	Sayegh
Cornea power	D_{ρ}	D_c	K
IOL power	D_{IOL}	D_{p}	L
EYE Power	$D_{\text{общ}}$	-	E

Table 1 Notations for Powers

Linear Measure	F67	F75	Sayegh	Sayegh-Reduced
Axial length	l	a	a	α
Focal length	f_2	-	f	φ
Postoperative ACD ("ELP")	k	k	e	ϵ
Second principal plane to IOL	H_2	-	l	λ

Table 2 Notations for Distances and Reduced Distances

Our notation eliminates the use of subscripts, and while using familiar eye-specific symbols, improves readability and makes explicit use of reduced distances. Powers are denoted by capital Latin letters usually related to the corresponding refracting surface designation. Distances are denoted by lower case Latin letters and the corresponding reduced distances (distance/index of refraction) by the corresponding Greek letter.

The F67 – F75 difference and some numerical comparisons

Using Sayegh's notation, F67 can be written as

$$L = \frac{1 - \left(\alpha - \varepsilon + \frac{\varepsilon}{n}\right)K}{(\alpha - \varepsilon)(\,1 - \,\varepsilon K)}$$

And F75 would read in same notation

$$L = \frac{1 - \alpha K}{(\alpha - \varepsilon)(\,1 - \,\varepsilon K)} \qquad \left((FS)_{(Fyodorov-Sayegh)}\right)$$

the difference Δ = F67-F75 can then be expressed as

$$\Delta = \frac{\epsilon K\left(\frac{n-1}{n}\right)}{(\alpha - \varepsilon)(1 - \epsilon K)} \qquad (\Delta)$$

Numerical examples are illustrated in Figure 1 where Δ is plotted as a function of the postoperative ACD or ELP. For example, using a 3.2 mm anterior chamber depth, the average reported by the authors, and for average values of keratometry and axial length, one finds a Δ just short of 2 D between F67 and F75 with F67 ~ F75 + 2 D. Given that the F75 is the correct derivation it is interesting to notice that the 67 paper comments that it is best to leave the patients with 1-2 D of myopia as they will have no accommodation left[1]. Note that Δ is a strictly increasing function of ELP and nearly doubles for an IOL placed in the bag of a "typical" eye.

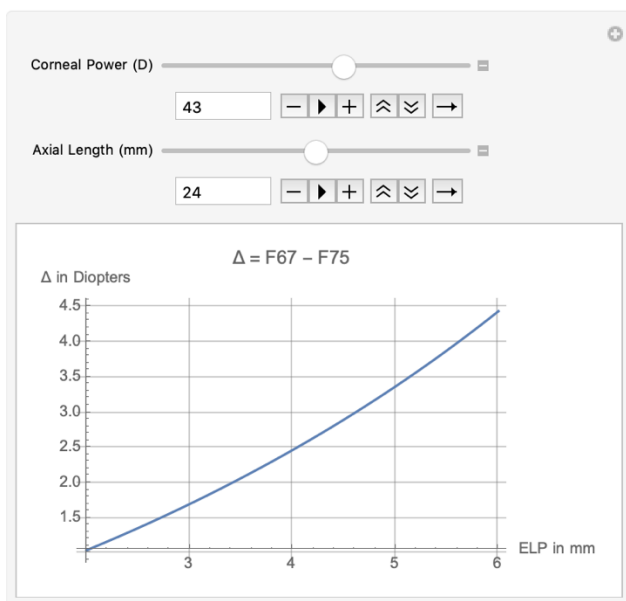

Figure 1 The difference in diopters between the IOL power prediction of Fedorov 1967 and Fyodorov 1975

1967 Fedorov and Kolinko approach and error. Sources and sources of error.

F67 relies on and cites findings from the 1950 edition of a Russian language physiological optics work by Kravkov (Кравков)[14]. The relations used are identical to those in Gullstrand 1909 and 1911[15]. The approach of Fedorov 1967 boils down to the following: Treat the pseudophakic eye as a "thick lens" with the first refractive surface being the cornea and the second the IOL. Express the power of that eye in two ways. One as the combination of the power of the cornea and that of the IOL, with the "thickness" being the postoperative ACD. The other as being the inverse of the focal length (i.e. the reduced second principal plane distance from the retina), which is expressed in terms of quantities including the axial length. Equating these two forms for the power of the eye yields the expression of the IOL power in terms of axial length, postoperative ACD, corneal power, and index of refraction of the aqueous/vitreous.

In more detail, what is done is the following.

1) Use the thick lens equation with the cornea and IOL as the two powers being combined and the (postoperative) anterior chamber depth as the "thickness" of the combination thick lens. The thick lens relation is referenced to Kravkov book p. 38. This approach results in an expression for the power of the eye as a function of the powers of cornea and IOL, the anterior chamber depth, and the index of refraction.

   Referring to tables 1 and 2 and the description above, we have 3 powers and 4 reduced lengths for a total of 7 variables. We can now write, in our compact notation, the two equations involving the power of the eye:

$$E = K + L - \varepsilon K L \qquad \left(\begin{matrix}(1)\\ -thick\ lens\ equation\end{matrix}\right)$$

   And:

$$E = \frac{1}{\varphi} \qquad (2) - (focal\ length - power\ relation)$$

2) Draw a diagram with 4 surfaces, A, B, C, D with A being the cornea plane and D the retina plane with C being the IOL plane and B the second principal plane of the eye (Figure 2).

$$D_{общ} = D_р + D_{иол} - \frac{k}{n} D_р D^1_{иол},$$

где $D_{общ}$ — общая рефракция глаза; $D_р$ — рефракция роговой оболочки; $D_{иол}$ — рефракция интраокулярной линзы; n — показатель преломления переднекамерной влаги и стекловидного тела (1,336).

Из формулы следует:

$$D_{иол} = \frac{D_{общ} - D_р}{1 - \frac{k}{n} D_р}. \quad (1)$$

Как видно на чертеже (рис. 1):

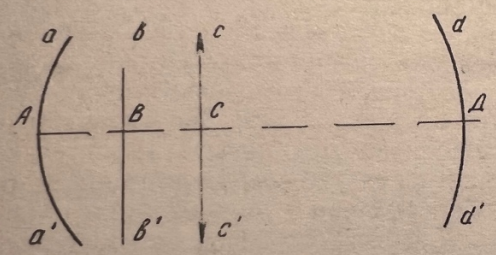

Рис. 1. Схематическое изображение основных оптических элементов глаза.

aa' — роговая оболочка; *бб'* — задняя главная плоскость глаза; *cc'* — интраокулярная линза; *dd'* — задняя стенка глаза; *AC* — глубина передней камеры (к); *AD* — длина оптической оси глаза (l); *BC* — расстояние от задней главной плоскости до линзы (H_2); *BD* — заднее фокусное расстояние (f_2).

[1] С. В. Кравков, 1950, с. 38.

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Figure 2 Adapted from Fedorov 67 with typos corrected in green

Simple geometry shows

$$BD = AD - AC + BC$$

Or,

$$f_2 = l - k + H_2$$

Where *l* is the axial length and k is the anterior chamber depth. In our compact notation:

$$f = a - e + l$$

Or, equivalently, after division by the index of refraction n:

$$\varphi = \alpha - \varepsilon + \lambda \quad (3)$$

3) In Fedorov 1967 article f_2 and H_2 are then expressed in terms of relationships known to relate the powers and distances to the principal planes' distances. Kravkov's book[14] has a discussion of this based partially on Gullstrand's contribution in von Helmholz' third edition of Physiological Optics (1909) and his 1911 monograph[15]. *The main error in Fedorov's paper* is introduced at this stage and the error is misinterpretation of a formula in Kravkov and *using a length instead of a reduced length* (Figure 3). It is worth noting that this 1967 paper by Fedorov is not cited in Fedorov 1975[2].

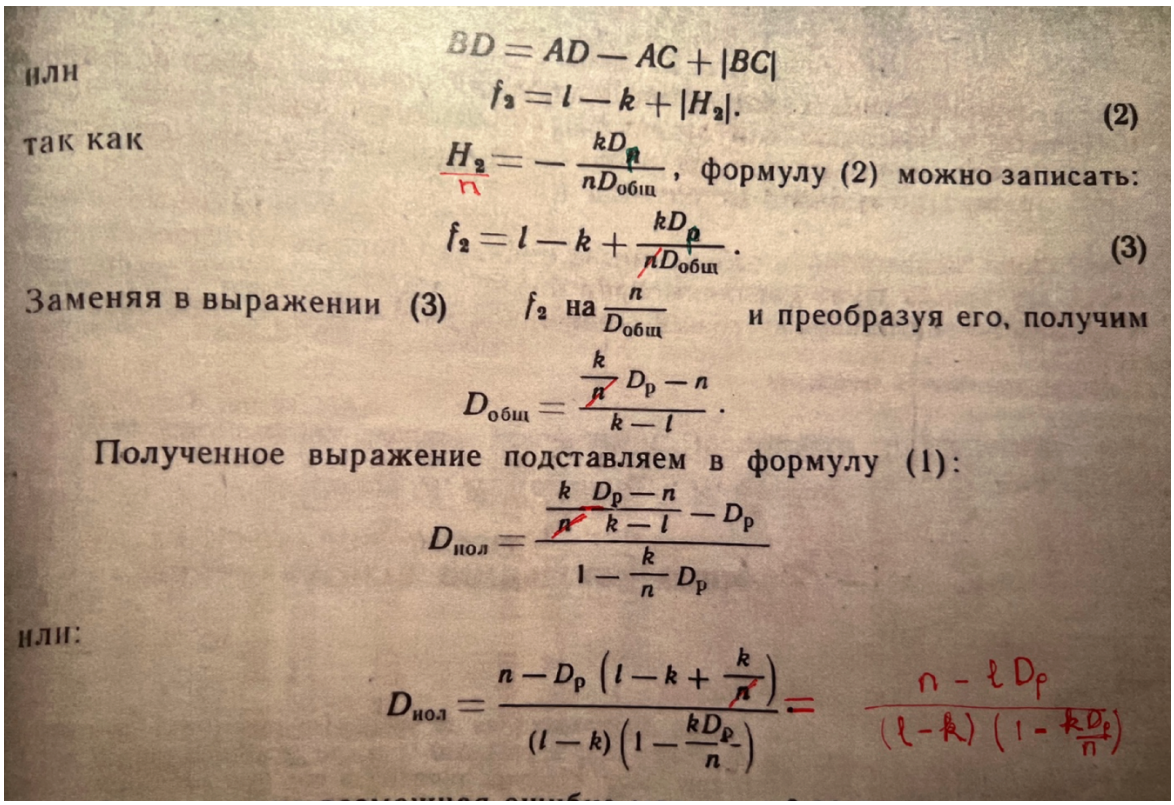
Finally, we express λ in terms of the fraction of ε:

$$\lambda = \frac{K}{E}\,\varepsilon \tag{4}$$

With four equations (1-4) we can, out of the 7 variables, eliminate 3, namely, E, φ, λ, leaving us with one relation between the 4 remaining variables, L, K, α and ε and we can thus express L in terms of the other 3 variables as follows:

$$L = \frac{1}{\alpha - \varepsilon}\frac{1 - \alpha K}{1 - \varepsilon K} \tag{FS}$$

This is trivially equivalent to F75 but not to F67. In Figure 3 we illustrate the error and the correction that leads to results equivalent to F75.



или

$$BD = AD - AC + |BC|$$

$$f_2 = l - k + |H_2|. \tag{2}$$

так как

$$\frac{H_2}{n} = -\frac{kD_p}{nD_{общ}},$$ формулу (2) можно записать:

$$f_2 = l - k + \frac{kD_p}{\cancel{n}D_{общ}}. \tag{3}$$

Заменяя в выражении (3) f_2 на $\frac{n}{D_{общ}}$ и преобразуя его, получим

$$D_{общ} = \frac{\frac{k}{\cancel{n}}D_p - n}{k - l}.$$

Полученное выражение подставляем в формулу (1):

$$D_{иол} = \frac{\frac{k}{\cancel{n}}\frac{D_p - n}{k - l} - D_p}{1 - \frac{k}{n}D_p}$$

или:

$$D_{иол} = \frac{n - D_p\left(l - k + \frac{k}{\cancel{n}}\right)}{(l - k)\left(1 - \frac{kD_p}{n}\right)} = \frac{n - lD_p}{(l - k)\left(1 - \frac{kD_p}{n}\right)}$$

Figure 3 Adapted from Fedorov 67 with typos corrected in green and errors in red

The thick lens equation in its modern form can be attributed to Gauss[16]. More recent treatments (prior to and following the Fedorov publications) can be easily found in the Optics literature, from monumental foundational work[17,18] to very useful practical manuals[19]. Prior to the 1967 article, reviews of the classic Born and Wolf Optics text were already appearing including in the biology literature.[20]

Discussion

Fedorov and Kolinko's foundational equation as formulated in 1967 is a simple yet profoundly insightful and influential approach to the calculation of the power of IOLs, yet it *is incorrect*. It has been praised and cited for decades by generations of leading ophthalmologists that proceeded to build on their legacy. The apparent lack of awareness or mention of the differences between the 1967 version and the 1975 is stunning! This may be due to the fact that the article was in Russian and published in the Russian literature. This is only partially reasonable since there was a significant interest in "monitoring" Russian publications at the height of the Cold War[21] and even decades earlier keen interest in the scientific Russian literature had already developed[22]. It is also worth mentioning that even current Russian Ophthalmology literature continues to refer to that paper as foundational[23].

Dogmatic views have sometimes persisted in Ophthalmology and in Medicine. Fairly recent examples from cataract surgery include the A "constant" that has "no units" (AAO Basic and Clinical Sciences, Lens and Cataract, various earlier editions, and persistent in recent quotes [24] despite clarifications [25]) and the "constant" 30 degrees rotation for total loss of astigmatism corrected by a toric IOL, and shown to be variable, implicitly by Felipe et al[26] and explicitly by Sayegh[27]. Various beliefs, rather than rigorous insight, continue to pervade the medical and ophthalmological communities including a rising belief that artificial intelligence methods are all powerful and can be a substitute for rigorously established scientific methodologies. It is imperative to debunk such naïve beliefs and make sure that the rising generation of ophthalmologists and vision scientists are schooled in the most rigorous intellectual tradition.

It is undeniable that the work by Fedorov and Kolinko constitutes one of the pillars of a paradigm shift that, along with high quality intraocular lenses delivered through smaller incisions and a number of other innovations, moved cataract surgery from a risk prone procedure with moderate outcomes to one of the most sophisticated and successful procedures in the history of Medicine[12,28]. This shift took place mainly in the last quarter of the 20th century and is ongoing. To continue building on this success it behooves us to reach a full assimilation of the process it took to get where we are and delve into the methods and their derivations. Maintaining a healthy centuries-old intellectual and scientific tradition is also likely to accelerate our future progress towards better results for our patients.

Fully understanding giants' mistakes, while standing on their shoulders, is a most valuable building block of our next glorious pyramid!

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